COST526 – Project PL1

Final Report

Optimization of tool shape in the tests aiming at identification of models describing rheological and mechanical properties of metallic alloys

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1. Introduction

The PL1 work group joins scientists from The Department of Computer Methods in Metallurgy, which belongs to the Faculty of Metallurgy and Materials Science of the Akademia Górniczo-Hutnicza, University of Science and Technology (AGH-UST). Main scientific topics of research in the Department include: optimization of tool shape in forging and extrusion processes; developement of constitutive models describing the state of the material during thermomechanical processing; multiscale analysis for simulation of microstructural phenomena and its influence on materials behavior and properties; laboratory tests for axisymmetrical and plane strain compression, tension and torsion and analysis of results based on the inverse techniquefor the evaluation of material constants in constitutive models.

Within the COST 526 Action, a dies design procedure based on optimization techniques, for semi-solid materials analysis, was elaborated. The objective of the analysis is development of constitutive models, as well as evaluation of material and process parameters. Forming of materials in semi-solid state is an efficient method of shaping. Thixoforming, which requires special rheocast microstructure (Figure 1), is particurarly effective semi-solid process. This method allows forming of materials with some special usefull properties and which, on the other hand, cannot be deformed in the typical metal forming conditions (Figure 2).

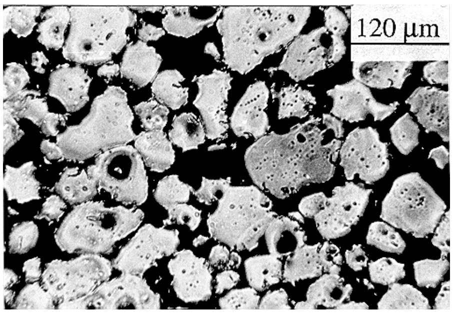

Fig. 1. Microstructure of the AlSi alloy in the semi-solid state (W. Lapkowski, J. Mat. Proc. Techn, 80-81, 1998, 463-468)

Fig. 2. From left to right: stock sample, forging made of lead, forgings made of AlCu4Mg: conventional forging (load 32kN), conventional forging (load 91kN), thixoforging (load 32kN) (W. Lapkowski, J. Mat. Proc. Techn, 80-81, 1998, 463-468)

2. Goal of the Project

Commonly used plastometric tests for flow stress determination are tension, torsion and compression. Beyond this, the ring compression is used to determine the friction coefficient. All these test are efficient and easy to perform for solids but, they are not efficient for semi-solid materials. In order to determine a proper constitutive model suitable for the simulation of the semi-solid metal deformation, the main goal of the project was to design the optimal experiment providing material characteristics and process parameters

A proposition of the test, which is based on the combined forward and backward extrusion, was analysed. Its schematic illustration is shown in Figure 3. The idea of the test is based on free flow of the material into upper and lower gaps. Measured parameters which provide information about boundary conditions and materials rheology are cups' walls heights and extrusion load. The die design was based on optimization of their shape and dimenstions (ram and die angles, upper and lower gap dimensions). Preliminary investigation involved simulation of metal flow and sensitivity analysis. The testing dies were designed, manufactured and assembled in the Gleeble 3800 simulator at the Institute for Ferrous Metallurgy in Gliwice, Poland.

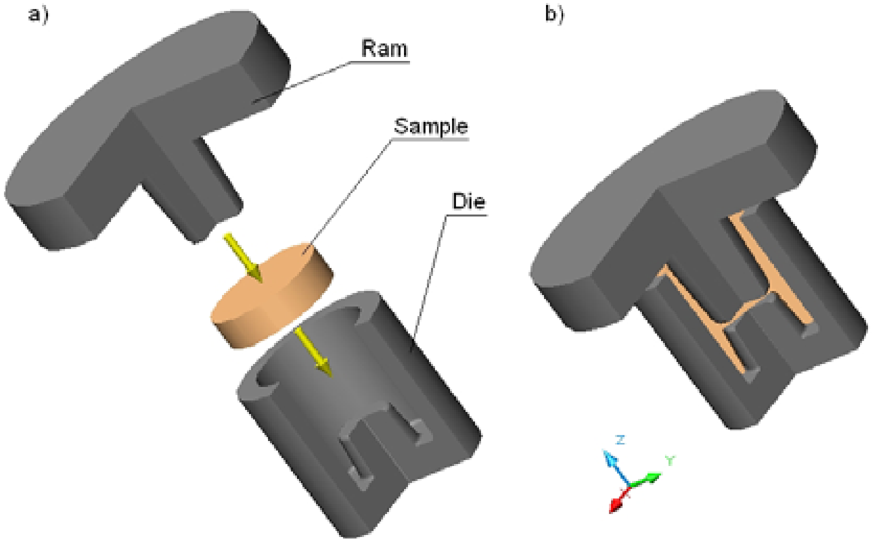

Fig. 3. Schematic view of the 'Double-cup' extrusion test: before – a), and after deformation – b).

3. Simulator, calibration, quality function and optimization algorithms, including assessment with respect to alternatives

3.1. Simulator

Forge2, Forge3 and CompAxi FEM codes were used for simulation of the investigated forward-backward extrusion process during design procedure, as well as for solution of the direct problem in the inverse analysis. The rigid-viscoplastic Norton-Hoff flow rule is applied in Forge2 and 3 codes:

$$\boldsymbol{\sigma} = 2K\left(T, \bar{\varepsilon}, \ldots\right)\left(\sqrt{3}\dot{\bar{\varepsilon}}\right)^{m-1} \dot{\boldsymbol{\varepsilon}} \tag{1}$$

where: σ – deviator stress tensor, *K* – consistency, *T* – temperature, $\bar{\varepsilon}$ – strain intensity, $\dot{\bar{\varepsilon}}$ – effective strain rate, $\dot{\boldsymbol{\varepsilon}}$ – strain rate tensor, *m* – strain rate sensitivity.

For the semi-solid materials analysis, the flow stress is given by the following equation:

$$\sigma_0 = K_s + \left(K\dot{\varepsilon}^m\right) \tag{2}$$

where: K – consistency, K_s – threshold consistency, m – strain rate sensitivity.
In hot forming of metals in solid state parameter m of 0.2 is assumed. For semi-solids this coefficient vary between 0.2 and 1. The consistency is close to 0 in the two phase region of temperatures. Coulomb friction model is used in the Forge 2 and Forge 3 codes.
The CompAxi code uses rigid-plastic finite element model. The friction model based on Treska law, combined with the sensitivity of friction on slip velocity as proposed by Chen and Kobayashi, is implemented in this code. The friction stress in this model is given by the following equation:

$$\tau = \mu p \frac{2}{\pi} \arctan\left(\frac{v_s}{a}\right) \tag{3}$$

where: μ – friction coefficient, p – normal pressure at the contact surface, v_s – slip velocity between the die and the workpiece, a – coefficient, which is approximately 2-3 orders smaller that the average slip velocity.

3.2. Objective functions

3.2.1. Dies design

The objective of the designed experiment is supplying information, which will allow accurate and reliable determination of the friction coefficient and the rheological parameters for materials deformed in the two-phase semi-solid conditions. Searching for the tool design, which involves flow of the material sensitive to friction conditions and to material's rheology, was performed and optimal die shape was designed. There were four variable parameters defined (Figure 4): two ram angles (α, β) and two gaps between the ram and the die (g_1, g_2). The task was to determine values of these parameters, which give the largest sensitivity of metal flow to rheological (consistency K) and friction (coefficient μ in the Coulomb friction model) parameters. The extrusion load and the ratio $H = h_1/h_2$ (as shown in Figure 1a, h_1 and h_2 represent height of the filling of the groove in the upper and the lower die, respectively) were the measured parameters, which are used to identification of friction and rheological models.

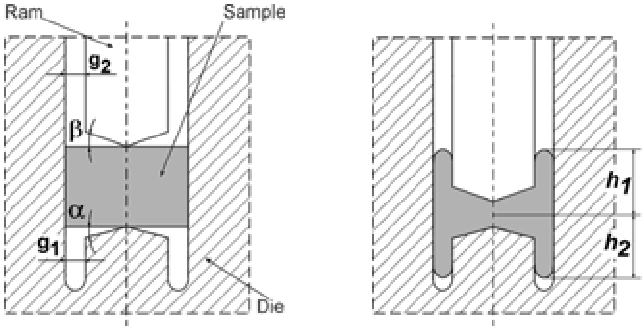

Fig. 4. Schematic illustration of dies parametrization for design procedure, dies and sample before (left) and after (right) the process

Searching for the largest sensitivity of the loads and the ratio $H = h_1/h_2$ to both friction and rheologicl parameters was expressed by following objective function:

$$\Phi = \sqrt{\left[\frac{\partial H(\mathrm{a})}{\partial \mu}\right]^2 + \left[\frac{\partial H(\mathrm{a})}{\partial K}\right]^2 + \left[\frac{\partial F(\mathrm{a})}{\partial K}\right]^2} \tag{4}$$

where: $\mathbf{a} = \{\alpha, \beta, g_1, g_2\}^T$

The search was constrained to $0 < \alpha < 30°$, $0 < \beta < 30°$, $2 < g_1 < 30$ mm, $2 < g_2 < 30$ mm. The following results were obtained for the goal function (1): $\alpha,\beta = 0°$ (at the boundary), $g_1{:}g_2 = 9{:}1$.

3.2.2. Sensiticity analysis

Design of the tests aiming at determination of material parameters or boundary conditions has to be based on the analysis of sensitivity of the parameters, which are measured in the test (load F, inner and outer radius of the ring after compression, lower and upper cup walls heights after extrusion), on the parameters, which are to be identified (friction coefficient μ and parameters in the strain hardening curve – hardening coefficient a and hardening exponent m). For comparison of the sensitivity of the measured parameters of the ring compression test and forward-backward extrusion test on the rheological and friction parameters, determination of the following coefficients was performed:

$$S_{ring-1} = \frac{1}{R_{in\_0}} \frac{\partial R_{in\_top}}{\partial \mu} \qquad S_{ring-2} = \frac{1}{R_{out\_0}} \frac{\partial R_{out\_top}}{\partial \mu}$$

$$S_{ring-3} = \frac{1}{R_{in\_0}} \frac{\partial R_{in\_centre}}{\partial \mu} \qquad S_{ring-4} = \frac{1}{R_{out\_0}} \frac{\partial R_{out\_centre}}{\partial \mu} \tag{5}$$

$$S_{extr-1} = \frac{1}{h_{10}} \frac{\partial h_1}{\partial \mu} \qquad S_{extr-2} = \frac{1}{h_{20}} \frac{\partial h_2}{\partial \mu}$$

where: h_1, h_2 – height of the filling of the gap in the upper and lower die in extrusion, respectively, R_{in}, R_{out}, – initial inner and outer radius, respectively. Index "0" refers to the average value of the considered parameter.

Beyond this, the sensitivity of loads to friction coefficient for ring compression and extrusion tests was evaluated. The sensitivity for both considered tests was defined as:

$$S_{load\_\mu} = \frac{1}{F_0} \frac{\partial F}{\partial \mu} \qquad S_{load\_a} = \frac{1}{F_0} \frac{\partial F}{\partial a} \qquad S_{load\_m} = \frac{1}{F_0} \frac{\partial F}{\partial m} \tag{6}$$

where: F_0 – average load during the test.

In general, the sensitivity of dimensions of rings after compression to friction coefficient is slightly smaller than that observed for forward-backward extrusion. Both investigated tests show similar sensitivity of loads to parameters of the flow stress equation. Sensitivity of loads to friction coefficient differs slightly for the two tests, it increases rapidly for large friction coefficients in the extrusion and an opposite tendency is observed in the ring compression.

3.2.3. Inverse analysis

Inverse algorithm, is based on assumption, that any deformation process can be described by the set of equations:

$$\mathbf{d} = F(\mathbf{x},\mathbf{p}), \qquad F : R^l \to R^r \tag{7}$$

where $\mathbf{d} = \{d_1,...,d_r\}$ – vector of measured output process parameters (loads monitored during the process and shape of the sample after process), $\mathbf{x} = \{x_1,...,x_k\}$ – vector of material model parameters (coefficients in the flow stress function and in the friction model), $\mathbf{p} = \{p_1,..., p_l\}$ – vector of process variables, describing the conditions of the process (strain rate, temperature of sample, die and surrounding), *r* – number of measured parameters, *k* – number of unknown parameters in the model, *l* – number of process variables.

The objective of the inverse analysis is an evaluation of optimum values of vector **x** components. It leads to minimization, with respect to the vector **x**, of the distance between vectors containing calculated and experimental values:

$$\Phi(\mathbf{x}) = \sum_{i=1}^{n} \beta_i [\mathbf{d}_i^c(\mathbf{x},\mathbf{p}_i) - \mathbf{d}_i^m]^2 \tag{8}$$

where: $\mathbf{d} = \{d_1^m,...,d_r^m\}$ - vector of measured data, $\mathbf{d} = \{d_1^c,...,d_n^c\}$ - vector of calculated data, β_i – weight factors (i = 1 .. n), *n* - number of sampling points. Measured data $\mathbf{d}^m$ are obtained from experiment and values $\mathbf{d}^c$ are calculated using numerical model of the direct problem.

Since identification of parameters of the friction model and the rheological model is the mutually dependent procedure, these processes have to be performed simultaneously. Thus, the objective function combining measurements and predictions of loads with measurements and predictions of the shape of the ring after compression, is used:

$$\Phi = \sqrt{\frac{1}{q}\sum_{j=1}^{q}\left[\frac{R_j^c(\mathbf{x}) - R_j^m}{R_j^m}\right] + \frac{1}{n}\sum_{i=1}^{n}\left[\frac{F_i^c(\mathbf{x}) - F_i^m}{F_i^m}\right]^2} \tag{9}$$

where: F^c, F^m – calculated and measured loads, respectively, *n* - number of sampling points for load measurements, R^c, R^m – calculated and measured inner and outer radius of the ring after compression, respectively, *q* - number of points, in which radius was measured.

Similar goal function, with the ring radiuses substituted by the height of the filling of the grooves, is used for the forward-backward extrusion:

$$\Phi = \sqrt{\frac{1}{q}\sum_{j=1}^{q}\left[\frac{H_j^c(\mathbf{x}) - H_j^m}{H_j^m}\right] + \frac{1}{n}\sum_{i=1}^{n}\left[\frac{F_i^c(\mathbf{x}) - F_i^m}{F_i^m}\right]^2} \tag{10}$$

where: F^c, F^m – calculated and measured loads, respectively, *n* - number of sampling points for load measurements, H^c, H^m – calculated and measured h_1/h_2 ratio (Figure 4) after extrusion, respectively, *q* - number of tests.

3.3. Experiment

The experimantal part of the project contained two steps: cold etrusion of copper samples for vlaidation of the process (comparison with ring compression test) and hot extrusion of AlSi9Mg alloy in semi-solid range of temperature. Experiments were performed on the Gleeble 3800 simulator (Figure 5) in Institute for Ferrous Metallurgy in Gliwice in Poland.

Fig. 5. Preparing of experiment (Gleeble 3800)

The cold extrusion was performed with samples with various dimensions: diameter 17.9 – 18.5 mm, height 5 – 7 mm. Two different rams diameters were used, 9.5 and 11 mm, with corresponding gaps g_2 = 4.2 mm i 3.5 mm. Samples were extruded with constant strain rate $\dot{\varepsilon}=1\mathrm{s}^{-1}$. In order to compare the value of friction coefficient and rheological model parameters evaluated from extrusion test with coefficient obtained from other methods, number of ring compression tests were performed in the same process conditions with rings made of the same material. Rings dimensions were following: outer diameter 12 – 18 mm, inner diameter 6 – 9 mm, height 4 – 6 mm. The samples were deformed with two different strain rates $\dot{\varepsilon}= 0.1\mathrm{s}^{-1}$ and $1\mathrm{s}^{-1}$. Obtained rings were cut and measured in order to evaluate the friction coefficient in the ring compression process using Inverse analysis.

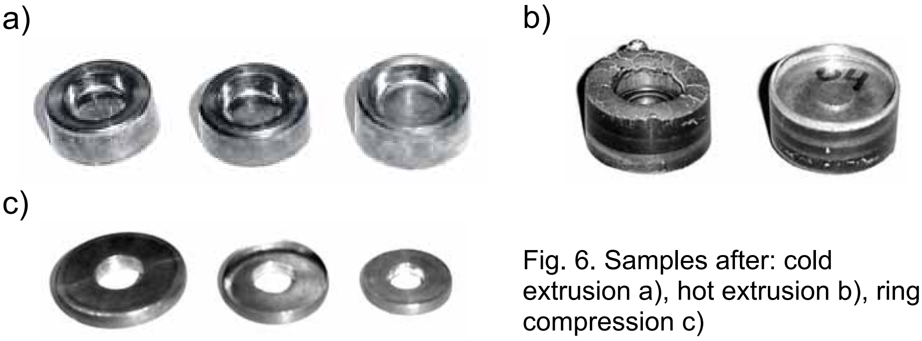

Fig. 6. Samples after: cold extrusion a), hot extrusion b), ring compression c)

3.4. Optimization (inverse analysis)

3.4.1. Cold processes

To analyse ability of the extrusion test to be used for rheology and boundary conditions testing, it was compared with the ring compression. The inverse analysis was performed to evaluate friction coefficient and rheological law parameters for two tests. The goal function (9) was used and OptyAxi Inverse environment for ring compression analysis. The measured and predicted ring radiuses were compared at various locations along the height of the sample. Optimization using the goal function (9) yielded the friction coefficient μ = 0.13, which gave very good agreement between measured and predicted shapes of samples. Similar approach was used in the inverse analysis of the extrusion process. The goal function (10) was used for determination of the friction coefficient with Forge2 FEM code called from external Simplex optimization procedure. Optimization yielded the friction coefficient μ = 0.18, which also gave very good agreement between measured and predicted shapes of samples. Larger value of friction coefficient during extrusion was explained as a result of higher contact pressures appearing on the contact surface between dies and deformed material. Simultaneously, inverse method based on the cost functions (9) and (10) allows identification of the rheological model. Following function was selected for description of the flow stress relation on strain:

$$\sigma = \sigma_0 + a\varepsilon^m \exp(-b\varepsilon) \tag{11}$$

Results of the inverse analysis for the coefficients σ_0, *a*, *b* and *m* are given in Table 1.

Tab. 1. Coefficients in equation (11) obtained from the inverse analysis

σ_0	*a*	*m*	*b*
103	320.8	0.288	0.7

Figure 7 shows comparison of measured loads with those calculated by the finite element code with the constitutive law based on equation (11) with the optimized coefficients. Good agreement is observed.

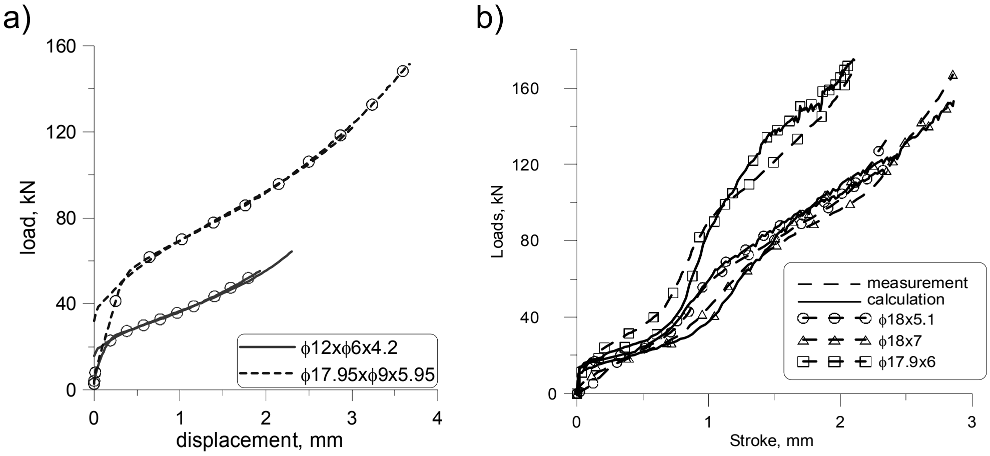

Fig. 7. Measured loads compared with loads calculated by the FEM code with the optimal parameters of constitutive law for copper in ring compression a) and extrusion b)

3.4.2. Hot process

Most common materials deformed in semi solid state are the Aluminium alloys, thus AlSi9Mg alloy was chosen to identify rheological and tribological model parameters using the developed inverse algorithm. This alloy is very difficult to deform using conventional metal forming processes. Chemical composition and properties of the investigated alloy are presented in table2.

Tab. 2. Chemical composition of AlSi9Mg aluminium alloy

Elem.	Al	Si	Cu	Mg	Mn	Fe
%	89.01	9.13	0.3	0.4	0.35	0.5
Elem.	Ti	Zn	Ni	Pb	Sn	Cr
%	0.05	0.2	0.02	0.02	0.006	0.01

The direct problem in the inverse analysis of the hot extrusion process was solved by the Forge2 finite element code, where the material model is described by the rigid-viscoplastic Norton-Hoff flow rule defined by equation (1). As it has already been mentioned, for semi-solid materials the coefficient *m* vary between 0.2 and 1. The consistency is close to 0 in the two phase region of temperatures. The flow stress is calculated as:

$$\sigma_0 = K_s + \left(K\dot{\varepsilon}^m\right) \tag{12}$$

where: *K* – consistency, K_s – threshold consistency, *m* – strain rate sensitivity.Relation of the parameters *K* and *m* on temperature is described by sigmoidal finctions:

$$K(T) = \frac{K_s - K_l}{1 + e^{\frac{T - T_K^{crit}}{dT_K}}} + K_l \tag{13}$$

$$m(T) = \frac{m_s - m_l}{1 + e^{\frac{T - T_m^{crit}}{dT_m}}} + m_l \tag{14}$$

where: K_s – solid material consistency, K_l – liquid material consistency, T_K^{crit} – critical temperature in whch consistency rapidly decreases, m_s – solid material sensitivity to strain rate, m_l – strain rate sensitivity for liquid material, T_m^{crit} – critical temperature for *m* parameter increase.

Optimized values of coefficients in equations (13) and (14) are presented in table 3 and forces calculated with this model parameters values are presented in figure 13 c) (solid lines with symbols). Results are compared with experimental curves and good agreement is observed. It can be concluded that the developed technique and method of interpretation of the results are the efficient tool for development of the constitutive law for semi-solid materials.

Tab. 3. Values of parameters of the rheological model obtained from the inverse analysis.

K_s	K_l	T_K^{crit}	dT_K	m_s	m_l	T_m^{crit}	dT_m
185.7	2.82	486.5	21.5	$8.1 \cdot 10^{-5}$	0.93	536.8	9.38

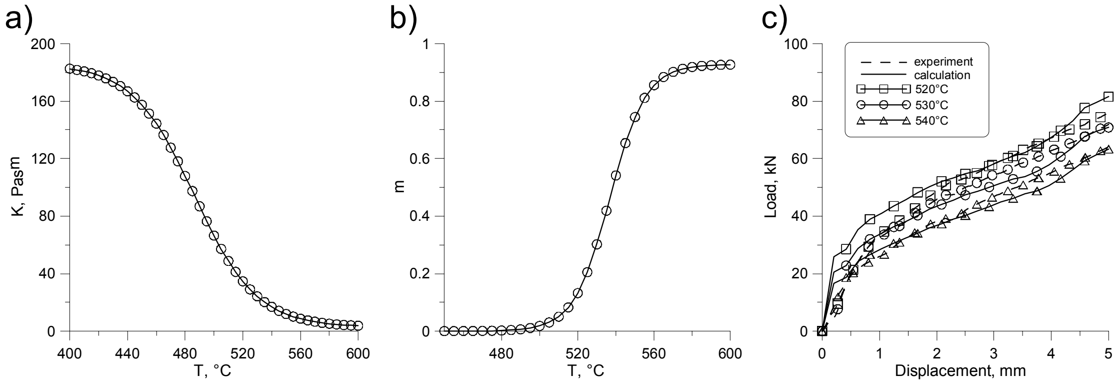

Fig. 8. *K* (a) and *m* (b) coefficients as functions of temperature; force vs. displacement curves, calculated with optimal rheological parameters (c)

4. Main scientific and technical outcome

The project concerning optimization of models describing rheological and mechanical properties of metallic alloys was based on three stages, each of them based on the modern scientific background in the field of application of optimization techniques to testing of materials and in the field of semi-solid deformation. During the first stage dies were designed using optimization methods and parameter sensitivity analysis was performed to obtain shape characterized by maximal sensitivity to materials rheology and friction conditions. Afterwards, dies were manufactured and cold extrusion tests were performed to validate theoretical assumptions concerning the sensitivity. Results of extrusion were compared to ring compression test, which is commonly used for identification of material and friction models. The inverse analysis was used for interpretation of both ring compression and extrusion. It was proved that in certain conditions the extrusion test is even more efficient than ring compression (e.g. high contact pressures, which are common in industrial processes). The third and essential part of the project contained experiments of hot extrusion using dies designed on the basis of earlier analyses. Then the inverse analysis was applied to identify rheological model parameters of material in semi-solid state.

The main scientific outcomes of the project are the methodology of identification of the rheological parameters on the basis of various tests and the values of these parameters for selected alloys in the semi-solid state.

Design of the equipment and experimental procedure for testing semi-solid materials is the main technical outcome of the project. Obtained results were satisfactory and proved that the 'double-cup' extrusion is an effective test for determination of material properties in hot conditions, which is often impossible using standard plastometric tests.

Beyond this several scientific articles was published:

1. A. Żmudzki, R. Kuziak, M. Papaj, M. Pietrzyk, Identification of friction model in extrusion, Obróbka Plastyczna Metali, 3, 2004, 69-78.
2. A. Żmudzki, J. Gawąd, J. Kusiak, M. Pietrzyk, Application of Sensitivity Analysis to Die Shape Design for Inverse Analysis of Two-Phase Materials. Proc. 7th Int. Conf. on Computational Plasticity COMPLAS VII, Barcelona, Spain, 2003, CD-ROM.
3. A. Żmudzki, M. Papaj, R. Kuziak, J. Kusiak and M. Pietrzyk, Optimum Die Shape Design for Evaluation of Material Properties. Proc. 6th Conf. on Material Forming ESAFORM, ed., V. Brucato, Salerno, 2003, 139-142

4. A. Żmudzki, J. Gawąd, M. Papaj, R. Kuziak, J. Kusiak, M. Pietrzyk, Proposition of experiment for evaluation of material properties of metal alloys deformed in semi-solid state, Proc. 7th Conf. on Material Forming ESAFORM, ed., S. Storen, Trondheim, 2004, 667-670.
5. M. Pietrzyk, A. Żmudzki, D. Szeliga, R. Kuziak, Sensitivity of Parameters of Forward-Backward Extrusion Test on Material Properties and Friction, Proc. Conf. Materials Science & Technology 2004, New Orleans, Louisiana, 2004, 403-411.
6. A. Żmudzki, M. Papaj, R. Kuziak, J. Kusiak, M. Pietrzyk, Validation of the direct-indirect extrusion test, designed for evaluation of flow properties of metal alloys deformed in semi-solid state, Proc. 10th International Conference METAL FORMING 2004, Kraków, Steel Grips, nb.3, 2004, 541-545.
7. A. Żmudzki, M. Pietrzyk, P. Kotrbácek, J. Horský, Various plastometric tests for semi solid materials and their numerical simulations, Proc. 10th International Conference METAL FORMING 2004, Kraków, Steel Grips, nb.3a, 2004, 735-739.
8. A. Żmudzki, M. Pietrzyk, R. Kuziak, M. Papaj, Identification of material modelfor aluminium alloy using forward-backward extrusion test, Proc. First Invited COST 526 Conf. APOMAT, eds., D. Büche, N. Hofmann, Morschach 2005, 183-192.
9. A. Żmudzki, J. Kusiak, Neural Networks based optimization in Inverse anlysis, Proc. First Invited COST 526 Conf. APOMAT, eds., D. Büche, N. Hofmann, Morschach 2005, 236-241.
10. A. Żmudzki, J. Gawąd, M. Papaj, R. Kuziak, J. Kusiak, M. Pietrzyk, Propozycja eksperymentu do wyznaczania parametrów reologicznych stopów metali odkształcanych w fazie półciekłej, Mat. 11 Konf. Informatyka w Technologii Metali KomPlasTech 2004, eds. F. Grosman, A. Piela., M. Pietrzyk, J. Kusiak, Zakopane, 2004, 83-90. (in Polish).
11. Rauch Ł., Talar J., Żak T., Kusiak J., Filtering of Thermomagnetic Data Curve Using Artificial Neural Network and Wavelet Analysis. Proc. ICAISC 2004 Conf., 2004, 1093-1098.
12. T. Zak, Ł. Rauch, J. Talar „Mathematical Processing of Termomagnetic Curves", 5th International Conference on Measurement, Smolenice, Slovakia, 2005.

5. Collaboration within COST 526

Collaboration with the TU Brno was a part of the project. Different experimental capabilities, which allow compression of semi-solid cylindrical samples, were the motivation of the collaboration. The data from hot upsetting test for cylindrical samples, performed in The Technical University of Brno (Czeh Rep.), has been used to verify the proposed model. The Inverse analysis of test results has been carried out to evaluate the model parameters,. Examined material was a tool carbon steel containing: 1.03%C, 0.22%Mn, 0.05%Ni, 0.19%Si. The Simplex algorithm was applied to solve the minimization problem, and FEM code was used to solve the direct problem in the inverse analysis. Achieved results confirm correct model behaviour, for both different types of alloys and their structures. The rheological parameters for tool carbon steel were determined. A short term technical visit of Petr Kotrbacek from Brno at AGH was completed. Joint paper [7] was published.

Collaboration with the Czech Academy of Sciences in Brno was conducted, as well. The objective was the optimal filtering of the thermo-magnetic data curves. New methods of filtering of experimental data curves, based on the artificial neural networks and the wavelet analysis were used. The obtained promising results were published in joint papers [11,12].