COST 526
“Automatic Process Optimization in Materials Technology”
(APOMAT)

Half-Yearly Report

To be sent to **V.Tesch@access.rwth-aachen.de** until **August 31, 2002**

1. Reporting Period	**1.1.2002 – 30.6.2002**
Project title	Forging Process Optimisation
Project leader Organization	Lionel FOURMENT CEMEF, Ecole des Mines de Paris
Main collaborators involved	Mehdi Laroussi

2. Funding Situation

Amount of money received specifically for COST	0 kEuros
Other resources partially used for the project	kEuros

3. International Collaboration
(mention group and type of work done in collaboration during the reporting period)

Participation in the Working Group Meeting in Saint-Dié des Vosges + project progress report

➢ YES

❒ NO

Group: “Bulk product processing”

Work done in collaboration: Submission of a French national funded project in the frame of Apomat, and organisation of it after its acceptance

4. Industry participation
(mention name of companies and work done in collaboration during the whole project)

Setforge, Sifcor (French forging companies) , Cetim (Technical Center of Mechanic Industry)

5. Meetings, visits, exchange of scientists, short-term scientific missions	**Location, date**
Project meeting	St Dié des Vosges, 21-22 May, 2002

Half-Yearly Report

6. Progress within the reporting period

(Not exceeding 3 pages, including tables and figures)

The work on forging optimisation has been carried out according to schedule. After having developed shape optimisation software in 2D, based of the FORGE2® simulation code, sensitivity analysis and a BFGS algorithm, it has been decided to extend the sensitivity analysis to 3D.

Several objective functions are to be considered (total forming energy, unfilling of final dies, folding defects, metallurgical quality, total strain, …) with respect to several process parameters (shape of the preforming dies, forging velocity, lubrication, initial part temperature, …). In order to get fast sensitivity analysis, the adjoint state method was developed to differentiate the discrete problem equations. Its extension to non-steady problems with contact evolution and remeshing has been studied.

Preliminary results have shown he feasibility of the approach; Now the computational time must be more precisely evaluated and the sensitivity analysis applied to more complex forging 3D problems.

(see proceeding of the Esaform conference in the following)

7. List of publications

a) Published

L. FOURMENT, M. LAROUSSI « The adjoint-state method for sensitivity analysis of non-steady problems: application to 3D forging », 5th ESAFORM conference, Krakòw, Poland, April 14-17

b) Submitted for publications

c) In preparation

L. FOURMENT**,** S. H. CHUNG, J.-L. CHENOT « Direct and adjoint differentiation methods for shape optimisation in non-steady forming applications with remeshing » soumis à Computers & Structures

THE ADJOINT-STATE METHOD FOR SENSITIVITY ANALYSIS OF NON-STEADY PROBLEMS APPLICATION TO 3D FORGING

ABSTRACT: Sensitivities of objective functions with respect to several parameters are calculated for metal forming applications. Based on the differentiation of the discrete problem equations, the adjoint state method is utilised for non-steady problems with large deformations, contact evolution but no remeshing. A backward analysis is carried out as a post-process, which requires additional storage of variables. The semi-analytical technique is used to compute the main derivatives. Finally, a simple forming case, for which an analytical solution is known, is treated in order to validate our sensitivities.

Key words: sensitivity analysis, adjoint state method, semi-analytical method, finite element method, forging

1 INTRODUCTION

The development of shape optimisation techniques for forming process design has resulted into several realisations in 2D. Most of them regard the optimisation of preforming tools for two-stage forging [1-5]. They are based on the direct differentiation technique and use a BFGS or similar algorithm for optimising the values of the shape parameters. Very different forging processes have been successfully investigated, so providing a significant aid to process design. Of course, all the problems have not been solved nor handled yet, however application of the same methods to 3D problem is quite attractive. Some results have already been obtained by Castro et al. [6], following exactly the same approach in spite of the increased complexity of 3D shape parameterisation.

Contrary to the 2D software, FORGE3® utilises an iterative solver that provides a very high efficiency for very large problems. In this case, the direct differentiation approach would provide a method which cost is proportional to the number of system resolution, e.g., to the number of optimisation parameters. So, for more than about five parameters, the sensitivity analysis would become more expensive than the direct one, which is not acceptable. Consequently, the adjoint state method for non-steady problems has been investigated in previous papers [7-9]. A preliminary work has been carried out in 2D in order to show the feasibility of the method [8,9], which cost is now proportional to the number of objective functions. Moreover, the sensitivity analysis can now be carried out as a post-processing operation, which presents obvious practical advantages.

After presenting the forming problem equations, this paper details the adjoint state method. Preliminary results of the method are given for a 3D non-steady application that does not require remeshing. The accuracy of the derivatives is evaluated by comparison with the analytical derivatives in a simple case. Finally, the additional storage required by the method is evaluated.

2 PROBLEM STATEMENT

2.1 Continuous problem statement

The material follows the Norton-Hoff law, under isothermal conditions. Inertia and gravity effects are neglected.

$$\mathrm{div}(v)=0\,,\ \sigma = s - p\mathbf{1}\,,\ p = -\tfrac{1}{3}\,\mathrm{trace}(\sigma) \qquad (1)$$

$$s = 2K(\sqrt{3}\dot{\bar{\varepsilon}})^{m-1}\dot{\varepsilon},\ \text{where } \dot{\bar{\varepsilon}} = \sqrt{\frac{2}{3}\dot{\varepsilon}:\dot{\varepsilon}} \qquad (2)$$

where v is the velocity field, $\dot{\varepsilon}$ is the strain rate tensor and σ the stress tensor. At the contact interface $\partial\Omega_c$, the “instantaneous” unilateral contact condition and the Norton friction law are written:

$$\begin{cases}(v - v_{tool}).n \le 0 \\ \tau = -\alpha K \|\Delta v_t\|^{q-1}\Delta v_t\end{cases} \quad \text{on } \partial\Omega_c \qquad (3)$$

where τ is the shear stress, Δv_t is the relative tangential velocity and v_{tool} is the tool velocity.

2.2 Discretisation

A P1+P1 velocity-pressure interpolation is used. The pressure is linearly interpolated like the velocity. A bubble then enriches the velocity interpolation in order to satisfy the compatibility condition.

$$x_h = \sum_{k=1}^{Nbnoe} X^k N^k\,,\ v_h = \sum_{k=1}^{Nbnoe} V^k \hat{N}^k\,,\ p_h = \sum_{k=1}^{Nbnoe} P^k N^k \qquad (4)$$

and then, $$\dot{\varepsilon}_h = \sum_{k=1}^{Nbnoe}\sum_{i=1}^{3} B^{ik} V^{ik} \qquad (5)$$

The following integration scheme is used:

$$\forall i = 0, N-1 \quad X_{i+1} = X_i + V_i \Delta t_i \qquad (6)$$

2.3 Finite element formulation

The contact condition being handled by a penalty method, at any time t_i, the discrete equations are:

$$\begin{cases} \forall k = 1,..\text{Nbnoe}, \forall i = 1,3 \\ \displaystyle\int_{\Omega_h} 2K\left(\sqrt{3}\dot{\bar{\varepsilon}}_h\right)^{m-1} \dot{\varepsilon}_h : B^{ik} dw_h - \int_{\Omega_h} p_h \, \mathrm{tr}\left(B^{ik}\right) dw \\ \displaystyle + \int_{\partial\Omega_{ch}} \alpha K \left\|\Delta v_{ht}\right\|^{q-1} \left(\Delta v_{ht}.e_i\right) \hat{N}^k ds_h \\ \displaystyle + \rho_{con} 1_{\partial\Omega_{pch}}(k) \left[\left(V^k - V_{tool}\right).n^k - \frac{\delta^k}{\Delta t}\right]^+ n_i^k = 0 \\ \displaystyle \forall k = 1,..\text{Nbnoe}, \quad \int_{\Omega_h} N^k \mathrm{div}\left(v_h\right) dw_h = 0 \end{cases} \qquad (7)$$

where ρ_{con} is a penalty coefficient and $[x]^+$ represents the positive part of x. It is summarised by:

$$\forall i = 0, N\text{-}1, \; R_i\left(X_i, W_i\right) = 0, \text{ with } W_i = \begin{pmatrix} V_i \\ P_i \end{pmatrix} \qquad (8)$$

3 SENSITIVITY ANALYSIS

3.1 Problem statement

Different measures, Φ, of the non-quality of the current metal forming process can be considered, like the total forming energy, the filling of the finishing dies, geometrical defects, metallurgical quality … The sensitivity analysis is then carried out with respect to either process parameters (forging velocity, friction coefficient, tool temperature …) or shape parameters (Bspline discretisation of the preform or the preforming tools), which are denoted by μ.

In the 3D frame, our main objective is to compute the various function sensitivities, rather than to solve the optimisation problem.

3.2 Adjoint state formulation

For each function Φ, a Lagrangian functional Λ is defined, with adjoint variables $\left(\lambda_i\right)_{i=0,N-1}$:

$$\Lambda(\mu, \lambda) = \Phi(\mu, X, W) + \sum_{i=0}^{N-1} {}^T\lambda_i R_i\left(\mu, X_i, W_i\right) \qquad (9)$$

where W_i satisfies the problem equations ($\forall i = 0, N-1$), so:

$$\frac{d\Lambda}{d\mu}(\mu) = \frac{d\Phi}{d\mu}(\mu) \qquad (10)$$

3.3 Calculation of the adjoint state

$\dfrac{d\Lambda}{d\mu}$ is calculated as follows:

$$\begin{aligned} \frac{d\Lambda}{d\mu}(\mu) &= \frac{\partial\Phi}{\partial\mu} + \sum_{i=0}^{N} \frac{\partial\Phi}{\partial X_i}\frac{dX_i}{d\mu} \\ &+ \sum_{i=0}^{N-1} \frac{\partial\Phi}{\partial V_i}\frac{dV_i}{d\mu} + \sum_{i=0}^{N-1} \frac{\partial\Phi}{\partial P_i}\frac{dP_i}{d\mu} \\ &+ \sum_{i=0}^{N-1} {}^T\lambda_i \left(\frac{\partial R_i}{\partial V_i}\frac{dV_i}{d\mu} + \frac{\partial R_i}{\partial P_i}\frac{dP_i}{d\mu} + \frac{\partial R_i}{\partial X_i}\frac{dX_i}{d\mu} + \frac{\partial R_i}{\partial \mu}\right) \end{aligned} \qquad (11)$$

$$\text{where:} \quad \forall i = 0, N \quad \frac{dX_i}{d\mu} = \frac{dX_0}{d\mu} + \sum_{j=0}^{i-1} \frac{dV_j}{d\mu}\Delta t_j \qquad (12)$$

The following variable is introduced:

$$\forall i = -1, N-1, \; \Gamma_i = \frac{\partial\Phi}{\partial X_N} + \sum_{j=i+1}^{N-1}\left(\frac{\partial\Phi}{\partial X_i} + {}^T\lambda_i \frac{\partial R_i}{\partial X_i}\right) \qquad (13)$$

The adjoint state variables are then chosen in order to eliminate the "implicit" derivatives $\dfrac{dV_i}{d\mu}$ and $\dfrac{dP_i}{d\mu}$:

$$\forall i = 0, N-1, \quad {}^T\lambda_i \frac{\partial R_i}{\partial W_i} = -\frac{\partial\Phi}{\partial W_i} - \Delta t_i \overline{\Gamma}_i \qquad (14)$$

where: $\overline{\Gamma}_i = \begin{pmatrix} \Gamma_i \\ 0 \end{pmatrix}$. Then:

$$\forall i = 0, N-1, \quad \lambda_i = -\left(\frac{\partial R_i}{\partial W_i}\right)^{-1\,T}\left(\frac{\partial\Phi}{\partial W_i} + \Delta t_i \overline{\Gamma}_i\right) \qquad (15)$$

λ_i is then backward computed, starting from i=N-1 down to i=0. Γ_i is also incrementally calculated as follows:

$$\begin{cases} \Gamma_{N-1} = \dfrac{\partial\Phi}{\partial X_N} \\ \forall i = N-1,0 \quad \Gamma_{i-1} = \Gamma_i + \dfrac{\partial\Phi}{\partial X_i} + {}^T\lambda_i \dfrac{\partial R_i}{\partial X_i} \end{cases} \qquad (16)$$

Once $\left(\lambda_i\right)_{i=0,N-1}$ have been evaluated, $\dfrac{d\Phi}{d\mu}$ is obtained by:

$$\frac{d\Phi}{d\mu}(\mu) = \frac{d\Lambda}{d\mu}(\mu) = \frac{\partial\Phi}{\partial\mu} + \sum_{i=0}^{N-1} {}^T\lambda_i \frac{\partial R_i}{\partial\mu} + \Gamma_{-1}\frac{dX_0}{d\mu} \qquad (17)$$

So, the adjoint state method is carried out in a backward way with respect to the original problem simulation. The sensitivity analysis is then a post-processing operation, after the simulation of the forming problem. It can be done independently, using stored information, X_i, W_i and contact data. The stiffness matrix used in (15), which is similar to that of the direct problem, could also be stored. It would require too much memory, so its re-calculation is preferred. At each time step, the method requires solving as many systems as objective functions.

3.4 *Additional storage*

A possible shortcoming of the adjoint state method is this requirement for an additional storage of variables. Its evaluation shows that it remains acceptable. At each time increment, the vectors X_i (3*Nbnode) and W_i (4*Nbnode) have to be stored, with double precision real arrays. It approximately occupies 56*Nbnode bytes. The memory used by the simulation of the direct problem is about 640*Nbnode bytes per time increment. However, this storage is only carried out every 10 or 20 time increments. The additional storage required by the adjoint state method is then comparable to and less than the one for the simulation itself. It is then considered quite acceptable.

3.5 *Actual derivative calculations*

The semi-analytical technique is used here, which means that the various derivatives, like $\frac{\partial R_i}{\partial X_i}$, are calculated by a finite difference scheme. It is verified [9] that the additional cost of the semi-analytical method is both independent on the number of parameters and on the number of nodes, so that the calculation of these derivatives is regarded as negligible with respect to the resolution cost.

4 NUMERICAL APPLICATION

The sensitivity analysis is applied to the upsetting of a cube between flat dies (see figure 1). The objective function is the total energy of the forming process:

$$\Phi = \int_{t_0}^{t_{fin}} \left(\int_{\Omega} K\left(\sqrt{3}\dot{\bar{\varepsilon}}\right)^{m+1} dw + \int_{\partial\Omega_t} \alpha K \left\| \Delta v_t \right\|^{q+1} ds \right) dt \qquad (18)$$

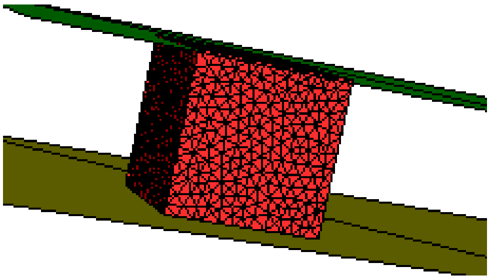

Fig. 1. 3D upsetting of a cube between flat dies

The width of the cube in the direction 1 and 2 is regarded as the optimisation parameter (see figure 2). Then, the workpiece coordinates in these direction, X_1 and X_2, are written as functions of the initial coordinates, x_1 and x_2, initial side length, a, and shape parameter, μ:

$$\forall i = 1,2, \quad X_i = x_i \left(1 + 2\frac{\mu}{a}\right) \qquad (19)$$

To validate our sensitivity analysis, tests are carried out in the Newtonian case (m=1) without friction ($\alpha = 0$). It is then possible to calculate an analytical expression of Φ and its derivative:

$$\Phi(\mu) = \sum_{i=0}^{N-1} \left(3K \left(\frac{v_{tool}}{h_{work,i}} \right)^2 a(a+2\mu)^2 \right) * \Delta t_i \qquad (20)$$

$$\frac{d\Phi}{d\mu} = \sum_{i=0}^{N-1} \left(12K \left(\frac{v_{tool}}{h_{work,i}} \right)^2 a(a+2\mu) \right) * \Delta t_i \qquad (21)$$

where $h_{work,i}$ is the distance between the dies at increment i.

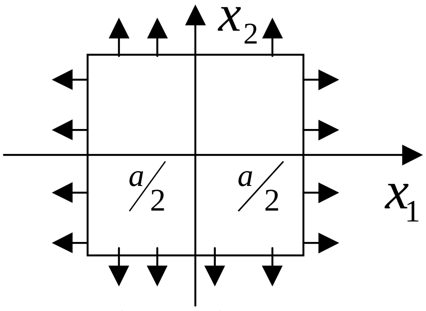

Fig. 2. Definition of the shape parameter.

The application is carried out with a mesh of 1510 nodes and 5157 tetrahedrons. The length of the cube edge is $a = 50\,\text{mm}$. The analytical derivatives are compared to the numerical ones in table 1. The agreement is very good, which validates the derivative calculations. Moreover, it is interesting to notice that the accuracy does not depend on the number of time increments, not showing any accumulation of discretisation error on this simple example.

Table 1. Comparison of analytical and numerical derivatives for the test case and for different number of time increments.

Number of time increments	$\left.\frac{d\Phi_{en}}{d\mu}\right\vert_{analytical}$	$\left.\frac{d\Phi_{en}}{d\mu}\right\vert_{numerical}$	Relative error
1	0,23152307	0,23152306	5.10^{-8}
5	1,15762463	1,15762457	5.10^{-8}
10	2,31527242	2,31527418	7.10^{-7}
15	3,47294336	3,47294332	5.10^{-8}
20	4,63063745	4,63063721	5.10^{-8}
100	23,1568925	23,1568913	5.10^{-8}

At each time increment the vectors X_i (3*Nbnode) and W_i (4*Nbnode) are stored, which occupies 84.8 Kbytes in our example.

5 CONCLUSIONS

The adjoint state method has then be successfully implemented for a non-steady forging problem, in the frame of a mixed velocity/pressure formulation. When the simulation does not require remeshing, the sensitivity analysis is shown to be quite accurate, whatever the number of time increments. The additional memory space requirement is also quite acceptable, and the analysis can be carried out as a post-processing operation. The method is complex but remains in the limits of what can reasonably be coded, more particularly using the semi-analytical approach.

Next step will be the application to more complex forging problems that require remeshing.

6 REFERENCE

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